down, and the pressure difference in the first region increases, this increase being more pronounced at the beginning of the heating process.

NOTATION

r, coordinate; t, time; T, temperature; P, pressure; ρ, density; λ, thermal conductivity; c, specific heat; a, thermal diffusivity; k, permeability; m, porosity; μ, viscosity; v, filtration rate; $\varkappa$, piezoconductivity; R(t), moving melting surface; L_2, latent heat of fusion of solid phase; T_m, melting point; z, z_1, z_2, self-similar variables; β, γ_1, γ_2, constants; A_1, A_2, constants of integration; ξ, u, auxiliary variables; α, mean specific heat; r_c, borehole radius; h, layer thickness.

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SOME PROBLEMS OF HEAT- AND MASS-TRANSFER THEORY SOLVABLE BY MEANS OF LAPLACE TRANSFORMATION

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The solution of a system of heat- and mass-transfer equations is obtained in Laplace transforms; formulas for finding the inverse transforms are given.

Consider the system of heat- and mass-transfer equations [1]

$$\frac{\partial u}{\partial t} = a_1 \frac{\partial^2 u}{\partial x^2} + k_1 \frac{\partial^2 v}{\partial x^2},$$
$$\frac{\partial v}{\partial t} = a_2 \frac{\partial^2 v}{\partial x^2} + k_2 \frac{\partial^2 u}{\partial x^2}, \tag{1}$$

where $a_1 > 0$; $a_2 > 0$; $k_1 > 0$; $k_2 > 0$; $a_1 a_2 > k_1 k_2$.

It is required to find the solution of this system for which boundedness conditions are satisfied: $u(x, t) = 0(e^{\lambda_1 x})$, $\lambda_1 > 0$; $v(x, t) = 0(e^{\lambda_2 x})$, $\lambda_2 > 0$; $(0 \leqslant x < \infty)$.

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$$u(x,\ 0)=0;\ v(x,\ 0)=0\ (0\leqslant x<\infty) \tag{2}$$

and boundary conditions of one of three forms:

$$\begin{cases} u(0,\ t)=\varphi_1(t), \\ v(0,\ t)=\varphi_2(t), \end{cases} \tag{3}$$

$$\frac{\partial u(0,\ t)}{\partial x}=\varphi_1(t),$$

$$\frac{\partial v(0,\ t)}{\partial x}=\varphi_2(t), \tag{4}$$

$$\begin{cases} E_1u(0,\ t)+F_1\dfrac{\partial u(0,\ t)}{\partial x}=\varphi_1(t),\ F_1\neq 0, \\ E_2v(0,\ t)+F_2\dfrac{\partial v(0,\ t)}{\partial x}=\varphi_2(t),\ F_2\neq 0. \end{cases} \tag{5}$$

Here $\varphi_1(t)$ and $\varphi_2(t)$ are polynomials of arbitrary order $(0 < t < \infty)$.

Laplace transformation is now performed, and the resulting equation solved, taking account of Eq. (2) and the boundedness condition, to give

$$\bar{u}(x,\ p)=B_1 e^{-\sigma_1\sqrt{p}x}+D_1 e^{-\sigma_2\sqrt{p}\,x},$$

$$\bar{v}(x,\ p)=B_2 e^{-\sigma_1\sqrt{p}x}+D_2 e^{-\sigma_2\sqrt{p}\,x},$$

where

$$\sigma_1=\sqrt{\frac{a_1+a_2+\delta}{2(a_1a_2-k_1k_2)}};\ \sigma_2=\sqrt{\frac{a_1+a_2-\delta}{2(a_1a_2-k_1k_2)}};$$

$$\delta=\sqrt{(a_1+a_2)^2+4(k_1k_2-a_1a_2)}=\sqrt{(a_1-a_2)^2+4k_1k_2}.$$

The coefficients B_1, B_2, D_1, and D_2 are expressed by the following formulas, in which either $i = 1$, $j = 2$, or $i = 2$, $j = 1$.

For boundary conditions of the form in Eq. (3)

$$B_i=\frac{1}{\delta}\left[\frac{1}{2}(a_j-a_i+\delta)\,\bar{\varphi}_i(p)-k_i\bar{\varphi}_j(p),\right],$$

$$D_i=\frac{1}{\delta}\left[\frac{1}{2}(a_i-a_j+\delta)\,\bar{\varphi}_i(p)+k_i\bar{\varphi}_j(p)\right].$$

For boundary conditions of the form in Eq. (4)

$$B_i=-\frac{1}{\sigma_1\delta}\left[\frac{1}{2}(a_i-a_j-\delta)\,\frac{\bar{\varphi}_i(p)}{\sqrt{p}}+k_i\,\frac{\bar{\varphi}_j(p)}{\sqrt{p}}\right],$$

$$D_i=\frac{1}{\sigma_2\delta}\left[\frac{1}{2}(a_j-a_i-\delta)\,\frac{\varphi_i(p)}{\sqrt{p}}-k_i\,\frac{\bar{\varphi}_j(p)}{\sqrt{p}}\right].$$

For boundary conditions of the form in Eq. (5)

$$B_i=\frac{1}{\delta F_1F_2\sigma_1\sigma_2}\left[\frac{1}{2}(a_j-a_i+\delta)\,\frac{E_j-F_j\sigma_2\sqrt{p}}{(\sqrt{p}-\alpha)(\sqrt{p}-\beta)}\,\bar{\varphi}_i(p)-k_i\,\frac{E_i-F_i\sigma_2\sqrt{p}}{(\sqrt{p}-\alpha)(\sqrt{p}-\beta)}\,\bar{\varphi}_j(p)\right],$$

$$D_i=\frac{1}{\delta F_1F_2\sigma_1\sigma_2}\left[\frac{1}{2}(a_i-a_j+\delta)\,\frac{E_j-F_j\sigma_1\sqrt{p}}{(\sqrt{p}-\alpha)(\sqrt{p}-\beta)}\,\bar{\varphi}_i(p)+k_i\,\frac{E_i-F_i\sigma_1\sqrt{p}}{(\sqrt{p}-\alpha)(\sqrt{p}-\beta)}\,\bar{\varphi}_j(p)\right],$$

where

$$\alpha = \frac{q-\sqrt{r}}{4F_1F_2\sigma_1\sigma_2};\ \beta = \frac{q+\sqrt{r}}{4F_1F_2\sigma_1\sigma_2};$$

$$q = \frac{1}{2}(E_1F_2+E_2F_1)(\sigma_1+\sigma_2)+(E_1F_2-E_2F_1)\frac{a_1-a_2}{2\delta}(\sigma_1-\sigma_2);$$

$$r = \frac{1}{4}\left[E_1F_2+E_2F_1+(E_1F_2-E_2F_1)\frac{a_1-a_2}{\delta}\right]^2\sigma_1^2+$$
$$+\frac{1}{4}\left[E_1F_2+E_2F_1+(E_2F_1-E_1F_2)\frac{a_1-a_2}{\delta}\right]^2\sigma_2^2+$$
$$+\frac{1}{2}\left[(E_1F_2+E_2F_1)^2+(E_1F_2-E_2F_1)^2\frac{(a_1-a_2)^2+8k_1k_2}{\delta^2}\right]\sigma_1\sigma_2.$$

It is not difficult to show that r > 0, so that α and β are real numbers. The expression $(E_i-F_i\sigma_i\sqrt{p})/[(\sqrt{p}-\alpha)(\sqrt{p}-\beta)]$ may be written in the form $A_i/(\sqrt{p}-\alpha)+B_i/(\sqrt{p}-\beta)$, where A_i and B_i are constants; in addition, $\overline{\varphi}_i(p)=\sum_{\nu=0}^{m_i}a_{i\nu}p^\nu$, $a_{i\nu}$ are constants. Therefore finding the solution u(x, t), v(x, t) reduces to calculating the inverse transforms of the following Laplace transforms:

1) $\overline{F}_j(p,\ k)=\exp(-k\sqrt{p})/p^{j+1}$, $k\geqslant 0$, $j=0,\ 1,\ 2\ldots$ for the conditions in Eq. (3);

2) $\overline{\Phi}_j(p,\ k)=\exp(-k\sqrt{p})/p^{j+\frac{3}{2}}$, $k\geqslant 0$; $j=0,\ 1,\ 2,\ \ldots$ for the conditions in Eq. (4);

3) $\overline{\Psi}_j(p,\ k,\ \gamma)=\exp(-k\sqrt{p})/[p^{j+1}(\sqrt{p}+\gamma)]$, $k\geqslant 0$; $\gamma\neq 0$; $j=0,\ 1,\ 2,\ \ldots$ for the conditions in Eq. (5).

The following operational formulas may be used to calculate $F_j(t,\ k)$, $\Phi_j(t,\ k)$, and $\psi_j(t,\ k)$

$$\overline{F}_j(p,\ k)\doteqdot F_j(t,\ k)=\frac{1}{j!}\sum_{\lambda=0}^{j}\left[(-1)^\lambda\binom{j}{\lambda}\left(\frac{k}{2}\right)^{2\lambda}t^{j-\lambda}J_\lambda(t)\right],$$

$$\overline{\Phi}_j(p,\ k)\doteqdot\Phi_j(t,\ k)=\frac{1}{j!}\sum_{\lambda=0}^{j}\left[(-1)^\lambda\binom{j}{\lambda}\left(\frac{k}{2}\right)^{2\lambda+1}J_{\lambda+1}(t)\right],$$

$$\overline{\Psi}_j(p,\ k,\ \gamma)\doteqdot\Psi_j(t,\ k,\ \gamma)$$
$$=\frac{1}{\gamma^2}\sum_{i=0}^{j}\frac{1}{\gamma^2(j-i)}[\gamma F_i(t,\ k)-\Phi_{i-1}(t,\ k)]+\frac{1}{\gamma^2(j+i)}\Psi(t,\ k,\ \gamma),$$

where

$$J_\lambda(t)=\frac{4^\lambda}{(2\lambda-1)k^{2\lambda-1}}t^{\lambda-1}\sqrt{\frac{t}{\pi}}\,e^{-\frac{k^2}{4t}}-\frac{2}{2\lambda-1}J_{\lambda-1}(t);$$

$$J_0(t)=\operatorname{erfc}\left(\frac{k}{2\sqrt{t}}\right),\ \Psi(t,\ k,\ \gamma)=$$
$$=\frac{1}{\sqrt{\pi t}}e^{-\frac{k^2}{4t}}-\gamma e^{k\gamma+\gamma^2t}\operatorname{erfc}\left(\frac{k}{2\sqrt{t}}+\gamma\sqrt{t}\right);$$
$$\Phi_{-1}(t,\ k)=\frac{d}{dt}\Phi_0(t,\ k)=\frac{1}{\sqrt{\pi t}}e^{-\frac{k^2}{4t}}.$$

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